Causal inference with interference

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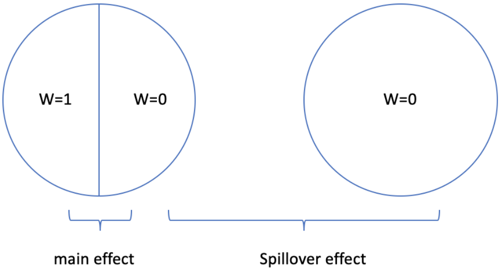
Outline

Definition of interference

Interference

Potential outcomes of a unit depend on its treatment and on the treatments of other local units.



Problems and Goals

Problems:

- Potential outcomes (SUTVA $\rightarrow$ SUTNVA)
- Assignment mechanisms (Unconfoundness $\rightarrow$ extension)
- Estimand (Biased naive estimand $\rightarrow$ extension)
- Estimation method (Propensity Score $\rightarrow$ Generalized PS)

Goals:

- Known network + unknown assignment mechanism
- Estimate: main effect and spillover effect

Notations

- Treatment $W_i \in \{0, 1\}$
- Outcome $Y_i \in \mathcal{Y}$
- Covariate $\mathbf{X}_i \in \mathcal{X}$;

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- Treatment $W_i \in \{0, 1\}$
- Outcome $Y_i \in \mathcal{Y}$
- Covariate $\mathbf{X}_i \in \mathcal{X}$; individual $\mathbf{X}_i^{ind} \in \mathcal{X}^{ind}$;neighborhood $\mathbf{X}_i^{neigh} \in \mathcal{X}^{neigh}$
- Undirected network $G = (\mathcal{N}, \mathcal{E})$
- For node i consider partitions $(i, \mathcal{N}_i, \mathcal{N}_{-i})$: $(W_i, \mathbf{W}_{\mathcal{N}_i}, \mathbf{W}_{\mathcal{N}_{-i}})$,$(Y_i, \mathbf{Y}_{\mathcal{N}_i}, \mathbf{Y}_{\mathcal{N}_{-i}})$

SUTNVA and potential outcomes

Assumption 1 No Multiple Versions of Treatment(Consistency)

$$Y_i = Y_i(\mathbf{W})$$

Assumption 2 Neighborhood Interference

Given a function $g_i : \{0,1\}^{N_i} \to \mathcal{G}_i$, $\forall i \in \mathcal{N}$, $\forall \mathbf{W}_{\mathcal{N}_{-i}}, \mathbf{W}'_{\mathcal{N}_{-i}}$ and $\forall \mathbf{W}_{\mathcal{N}_i}, \mathbf{W}'_{\mathcal{N}_i}$,$g_i(\mathbf{W}_{\mathcal{N}_i}) = g_i(\mathbf{W}'_{\mathcal{N}_i})$, the following equality holds:

$$Y_i(W_i, \mathbf{W}_{\mathcal{N}_i}, \mathbf{W}_{\mathcal{N}_{-i}}) = Y_i(W_i, \mathbf{W}'_{\mathcal{N}_i}, \mathbf{W}'_{\mathcal{N}_{-i}})$$

Potential outcomes

$$Y_i(w, g) = Y_i(W_i = w, G_i = g)$$

for a subset of nodes $V_g = \{i : g \in \mathcal{G}_i\}$ where $G_i = g_i(\mathbf{W}_{\mathcal{N}_i})$

Assignment mechanism

Assignment mechanism

$$P(\mathbf{W},\mathbf{G}|\mathbf{X},\{\mathbf{Y}(w,g),w=0,1;g\in\mathcal{G}\})=$$
$$\begin{cases} P(\mathbf{W}|\mathbf{X},\{\mathbf{Y}(w,g),w=0,1;g\in\mathcal{G}\}) & \text{if} \quad \mathbf{G}=g(\mathbf{W}) \\ 0 & \text{otherwise} \end{cases}$$

Assumption 3 Unconfoundness of Individual and Neighborhood Treatment

$$Y_i(w,g) \perp\!\!\!\perp W_i, G_i|\mathbf{X}_i \ \forall w\in\{0,1\}, g\in\mathcal{G}_i, \forall i$$

Causal estimands

Main effect $\tau(g)$ and overall main effect τ

$$\tau(g) = E[Y_i(W_i = 1, G_i = g) - Y_i(W_i = 0, G_i = g)|i \in V_g]$$
$$\tau = \sum_{g \in \mathcal{G}} \tau(g) P(G_i = g)$$

Spillover effect $\delta(g; w)$ and overall spillover effect $\Delta(w)$

$$\delta(g; w) = E[Y_i(W_i = w, G_i = g) - Y_i(W_i = w, G_i = 0)|i \in V_g]$$
$$\Delta(w) = \sum_{g \in \mathcal{G}} \delta(g; w) P(G_i = g)$$

Total effect TE

$$\sum_{g \in \mathcal{G}} E[Y_i(W_i = 1, G_i = g) - Y_i(W_i = 0, G_i = 0)|i \in V_g] P(G_i = g)$$

$$= \tau + \Delta(0)$$

Generalized propensity score

$$
\begin{aligned}
\psi(w;g;x) &= P(W_i = w, G_i = g|\mathbf{X}_i = \mathbf{x}) \\
&= P(G_i = g|W_i = w, \mathbf{X}_i^g = \mathbf{x}^g)P(W_i = w|\mathbf{X}_i^w = \mathbf{x}^w) \\
&= \lambda(g;w;x^g)\phi(w;x^w)
\end{aligned}
$$

- Joint propensity score: $\psi(w;g;x)$
- Neighborhood propensity score:$\lambda(g;w;x^g)$
- Individual propensity score: $\phi(w;x^w)$

Assumption 1-3

Assumption 1 No Multiple Versions of Treatment(Consistency)

$$Y_i = Y_i(\mathbf{W})$$

Assumption 2 Neighborhood Interference

Given a function $g_i : \{0,1\}^{N_i} \rightarrow \mathcal{G}_i$, $\forall i \in \mathcal{N}$, $\forall \mathbf{W}_{\mathcal{N}_{-i}}, \mathbf{W}'_{\mathcal{N}_{-i}}$ and $\forall \mathbf{W}_{\mathcal{N}_i}, \mathbf{W}'_{\mathcal{N}_i}$,$g_i(\mathbf{W}_{\mathcal{N}_i}) = g_i(\mathbf{W}'_{\mathcal{N}_i})$, the following equality holds:

$$Y_i(W_i, \mathbf{W}_{\mathcal{N}_i}, \mathbf{W}_{\mathcal{N}_{-i}}) = Y_i(W_i, \mathbf{W}'_{\mathcal{N}_i}, \mathbf{W}'_{\mathcal{N}_{-i}})$$

Assumption 3 Unconfoundness of Individual and Neighborhood Treatment

$Y_i(w,g) \perp\!\!\!\perp W_i, G_i | \mathbf{X}_i \ \forall w \in \{0,1\}, g \in \mathcal{G}_i, \forall i$

Properties of GPS

Properties

- **Balancing:**
  $P(W_i, G_i|\mathbf{X}_i, \psi(w; g; \mathbf{X}_i)) = P(W_i, G_i|\psi(w; g; \mathbf{X}_i))$
- **Conditional Unconfoundedness(joint)**
  $Y_i(w, g) \perp\!\!\!\perp W_i, G_i|\psi(w; g; \mathbf{X}_i)\ \forall w \in \{0, 1\}, g \in \mathcal{G}_i$
- **Conditional Unconfoundedness**
  $Y_i(w, g) \perp\!\!\!\perp W_i, G_i|\lambda(g; w; \mathbf{X}_i^g), \phi(1; \mathbf{X}_i^w)\ \forall w \in \{0, 1\}, g \in \mathcal{G}_i$

GPS estimator

$$E[E[Y_i|W_i = w, G_i = g, \psi(w; g; \mathbf{X}_i)]|W_i = w, G_i = g]$$
$$E[E[Y_i|W_i = w, G_i = g, \lambda(g; w; \mathbf{X}_i^g), \phi(1; \mathbf{X}_i^w)]|W_i = w, G_i = g]$$

Estimating procedure

Subclassification and GPS

1. Subclassification on $\phi(1;\mathbf{X}_i^w)$
   1. Estimate $\phi(1;\mathbf{X}_i^w)$: logistic regression $W_i \sim \mathbf{X}_i^w$
   2. Predict $\phi(1;\mathbf{X}_i^w)$
   3. Subclassify J subclasses B_j by $\phi(1;\mathbf{X}_i^w)$ where $\mathbf{X}_i^w \perp\!\!\!\perp W_i | i \in B_j$
2. Within B_j, estimate $\mu_j(w,g) = E[Y_i(w,g)|i \in B_j^g]$ where $B_j^g = V_g \cap B_j$
   1. Estimate $\lambda(g;w;\mathbf{X}_i^g)$: $G_i \sim W_i + \mathbf{X}_i^g$
   2. Estimate outcome model $Y_i \sim \lambda(g;w;\mathbf{X}_i^g) + W_i + G_i$
   3. Predict $Y_i(w,g)$
   4. Estimate $\hat{\mu}_j(w,g) = \sum_{i \in B_j^g} \hat{Y}_i(w,g)/|B_j^g|$
3. Estimate $\hat{\mu}(w,g) = \sum_{j=1}^J \hat{\mu}_j(w,g)\pi_j^g$ where $\pi_j^g = |B_j^g|/v_g$

Notice: $\mu(w,g) = E[Y_i(w,g)|i \in V_g], \forall w \in \{0,1\}, g \in \mathcal{G}$

Simulation

A friendship network from Facebook is obtained from Stanford Network Analysis Project(SNAP). It consists of 10 egocentric networks of over 4000 nodes and 80000 edges.

1. Individual covariates $X_i^{ind} = (Age_i, Gender_i)$
2. Neighborhood covariates $X_i^{neighbor} = (\frac{\sum_{j \in V_i} Age_j}{N_i}, \frac{\sum_{j \in V_i} Gender_j}{N_i}, N_i)$

Simulate 2 scenarios on this real network

1. $Z \perp\!\!\!\perp G | X^{ind}$
2. $Z \perp\!\!\!\perp G | X^{ind}, X^{neighbor}$

Data Generation Mechanism

Assignment Generation Mechanism

1. $logit(P(Z_i = 1)) = -1.5 + 1.2gender_i + 0.3age_i$
2. $logit(P(Z_i = 1)) = -4.5 + 0.4gender_i + 0.1age_i + 2.5friends.gender_i + 0.3friends.age_i$

Outcome Generation Mechanism

$$Y_i(z, g) \sim N(\mu(z, g, X_i^{ind}, X_i^{neighbor}), 1)$$

$$\mu(z, g, X_i^{ind}, X_i^{neighbor}) = 15 - 7I(\phi(1, X^{ind}) \geq 0.85) - 15z + 3zI(\phi(1, X^{ind}) \geq 0.85) + \delta g$$

Results

Comparison Procedure

1. A simple difference in mean outcome of treated and untreated.
2. A regression estimator on individual treatment Z_i & covariates
3. Subclassification on individual propensity score & covariates

Scenarios	δ	1	2	3	GPS
$Z \perp\!\!\!\perp G \mid X^{ind}$	low	-2.3	-0.58	-0.37	-0.02
	med	-2.46	-0.60	-0.41	-0.05
	high	-2.52	-0.53	-0.38	-0.01
$Z \perp\!\!\!\perp G \mid X^{ind}, X^{neighbor}$	low	-2.47	-1.80	-1.90	-0.7
	med	-2.78	-2.06	-2.18	-0.69
	high	-2.99	-2.23	-2.37	-0.69

Table 1: Bias of Estimators