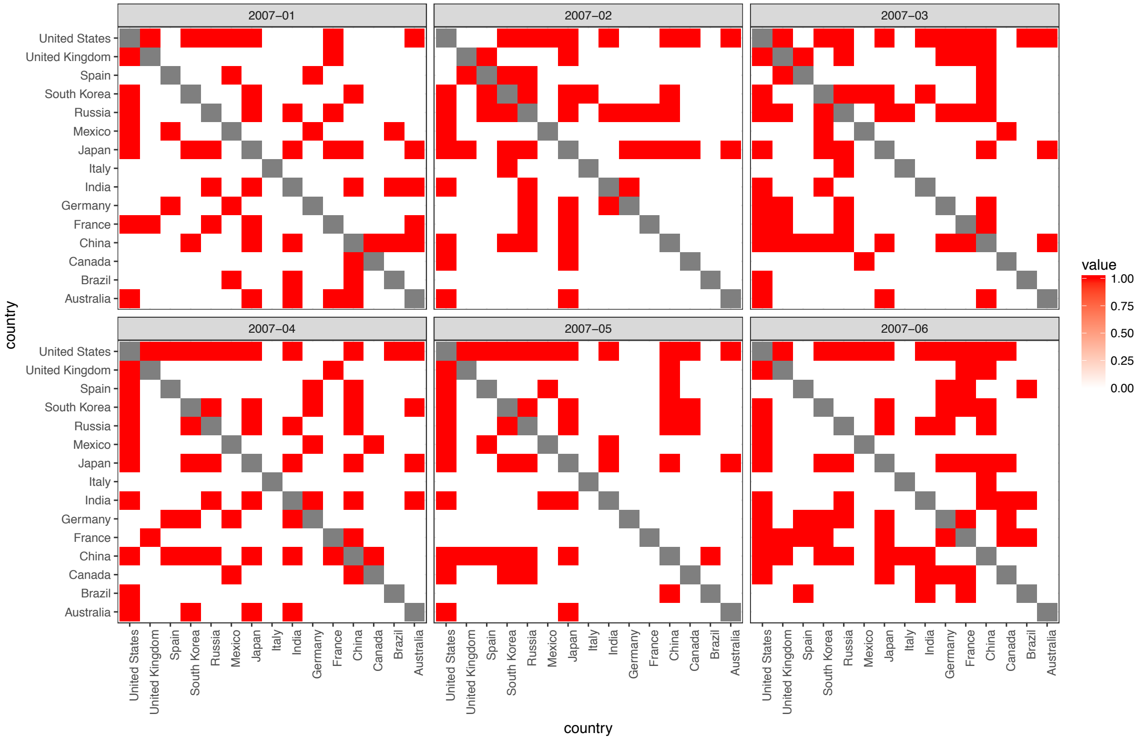
Modeling dynamic international relationship



- Challenges:
  - Binary data with 21.7% non-zero entries
  - Each $y_{ij,t}$ is a time series
  - Network structure
- Goal: Capturing the pattern

- Latent space model

$$y_{ij,t}|\pi_{ij,t} \sim Bernoulli\{\pi_{ij,t}\}$$

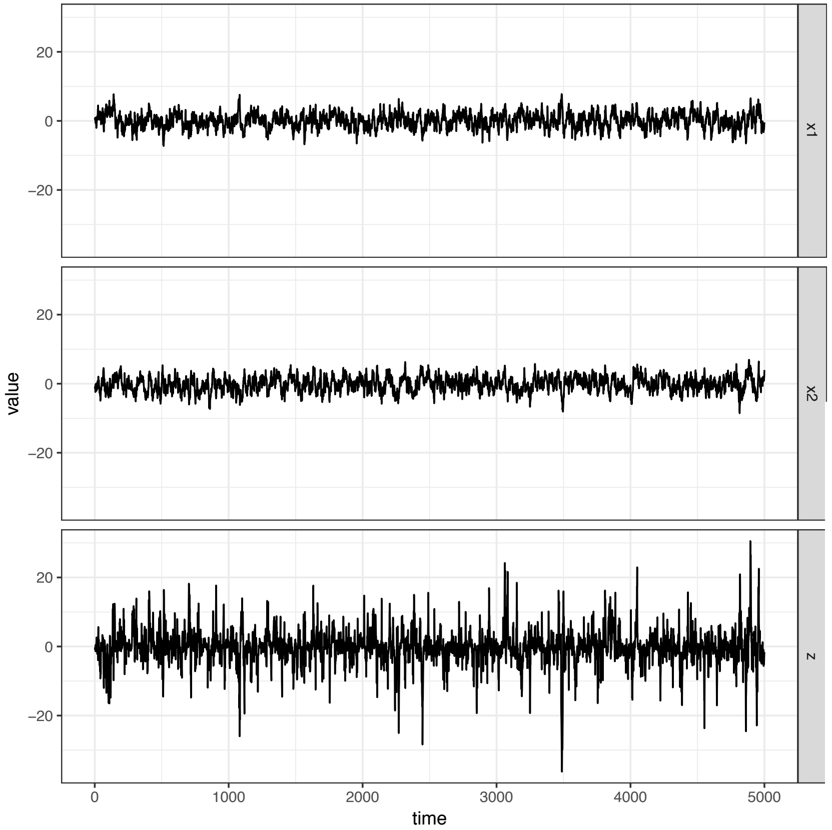
$$\pi_{ij,t} = \{1 + e^{-S_{ij,t}}\}^{-1}$$

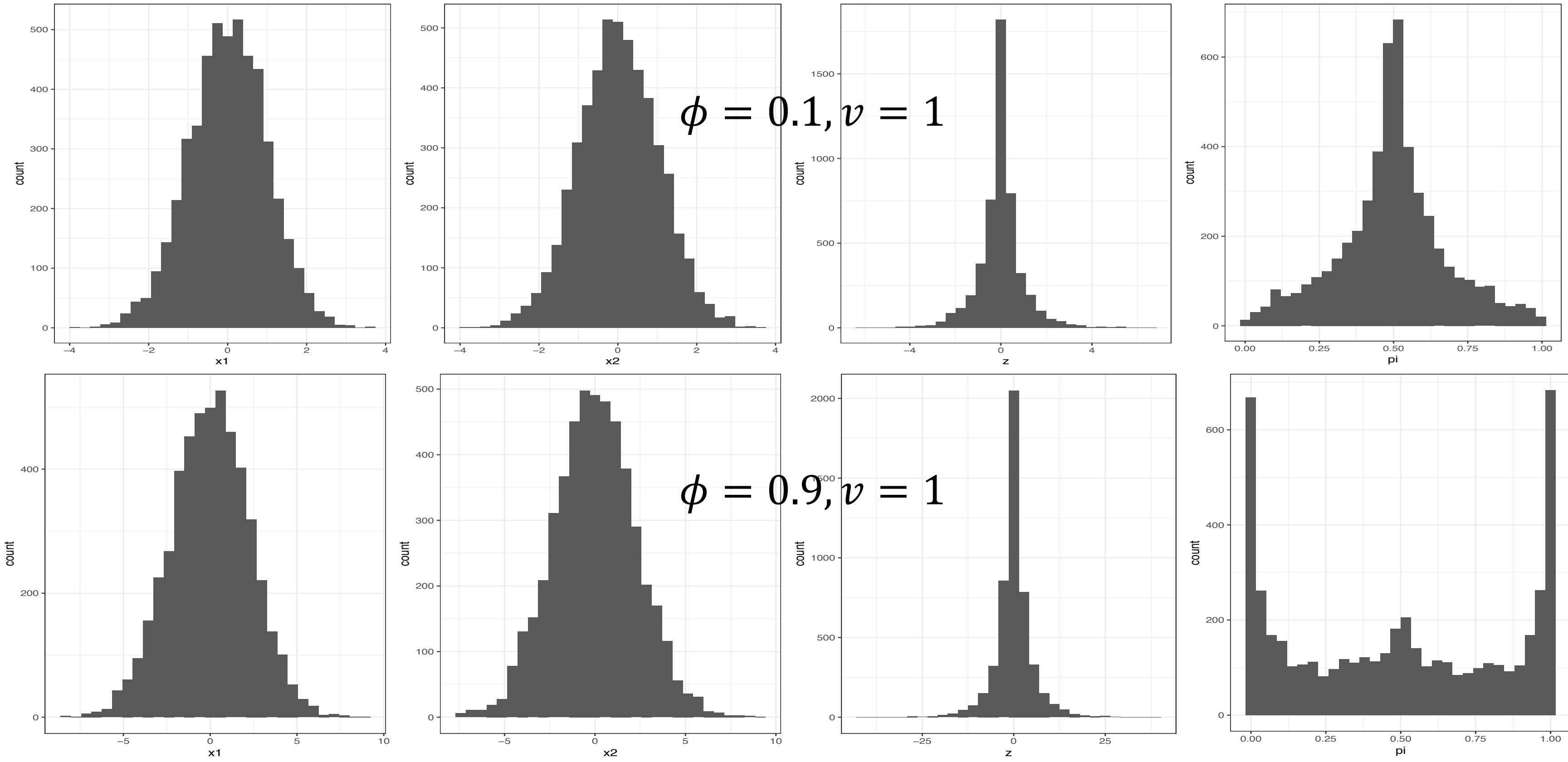
$$S_{ij,t} = \mu_t + x_{i,t}^T x_{j,t}$$

- Latent factor $x_{i,t} = \{x_{i1,t}, \ldots, x_{iH,t}\}^T$
- Stationary AR(1) process μ_t $x_{ih,t}$
- product of two AR(1) process

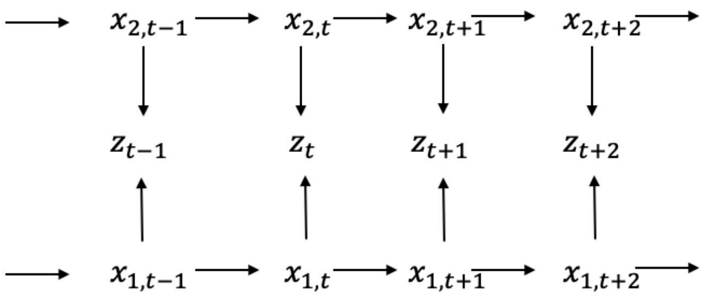
Properties of the product of two AR(1) process

$$z_t = x_{1t}x_{2t}$$





- Stationary
- Not Markov



- Cannot identify x_1 and x_2 , but z is identifiable

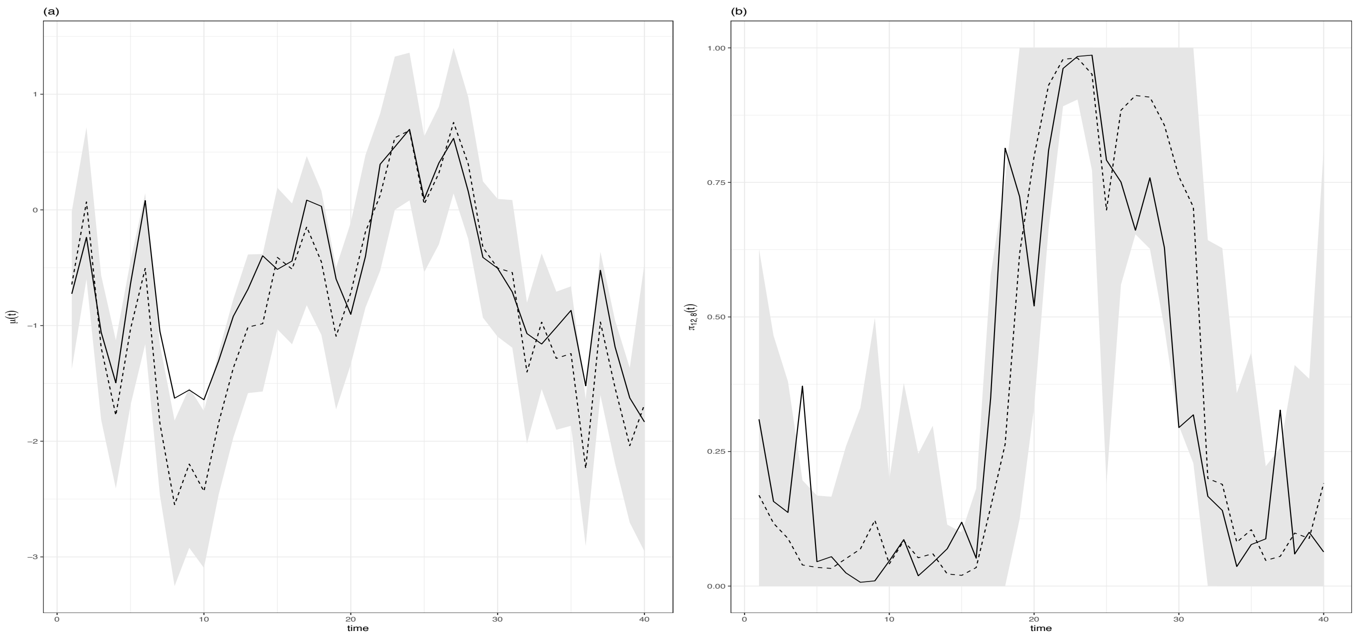
Sampling scheme and applications

- Gibbs Sampling with Polya-Gamma data augmentation

$$logit \pi_{ij} = \mu + x_i^T x_j$$

- From the joint distribution of AR(1) process

$$\mu \sim MVN(0, s\Phi_\mu), x_{ih} \sim MVN(0, s\Phi_x)$$



- Choice of $H, \phi_\mu, v_\mu, \phi_x, v_x$

H: large for better model fitting; add shrinkage prior
s: reasonable small range
ϕ: relative small for model fitting