Interpretation on $\hat{\beta}$s.

1.

Start from a SLR

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$$

1.(a)

If x_1 : continuous

$$\hat{y}' = \hat{\beta}_0 + \hat{\beta}_1 (x_1 + 1)$$
$$= \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_1 \quad \Rightarrow \quad \hat{\beta}_1 = \hat{y}' - \hat{y}.$$
$$= \hat{y} + \hat{\beta}_1$$

As $x_1 \uparrow$ by 1 unit, $y \uparrow$ by $\hat{\beta}_1$.

1.(b).(i)

If x_1 : factor. Take k levels ($k-1$ dummy var)

$$y \sim 1 + x_1$$

$$\Rightarrow \quad \hat{y} = \hat{\beta}_0 + \hat{\beta}_{1,1} x_{1,1} + \hat{\beta}_{1,2} x_{1,2} + \cdots + \hat{\beta}_{1,k-1} x_{1,k-1}$$

$x_{1,1} = I(x_1 = 1)$ indicator / dummy var.

$\hat{\beta}_0$: if $x_{1,1} = \cdots = x_{1,k-1} = 0$. * each observation can only take 1 level

$\hat{y}' = \hat{\beta}_0 + \hat{\beta}_{1,1}$: if $x_{1,1} = 1$, $x_{1,2} = \cdots = x_{1,k-1} = 0$.

$$\hat{\beta}_{1,1} = \hat{y}' - \hat{\beta}_0$$

$\hat{\beta}_{1,1}$: If x_1 is level 1, compared to baseline,

$y \uparrow$ by $\hat{\beta}_{1,1}$

1.(b)(ii).

If x_1 factor. k levels (k indicator)

$$\hat{y} = \hat{\beta}_{1,1} x_{1,1} + \cdots + \hat{\beta}_{1,k} x_{1,k}$$

$\hat{\beta}_{1,1}$: $x_{1,1} = 1$, $x_{1,2} = \cdots = x_{1,k} = 0$.

2. MLR. multiple predictors w/o interaction.

(hold other constant)

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon.$$

(a) If x_1 is continuous,

$$\hat{y}' = \hat\beta_0 + \hat\beta_1(x_1+1) + \hat\beta_2 x_2$$ holding constant

$$= \hat\beta_0 + \hat\beta_1 x_1 + \hat\beta_2 x_2 + \hat\beta_1$$

$$= \hat{y} + \hat\beta_1$$

$$\hat\beta_1 = \hat{y}' - \hat{y}.$$

Holding other var constant, $x_1 \uparrow 1$ unit $\Rightarrow \hat{y} \uparrow \hat\beta_1$

(b) If x_1 is factor with k levels.

$$\hat{y} = \hat\beta_0 + \hat\beta_{1,1} x_{1,1} + \dots + \hat\beta_{1,k-1} x_{1,k-1} + \hat\beta_2 x_2.$$

$\hat\beta_0$: $x_{1,1} = \dots = x_{1,k-1} = 0$, $x_2 = 0$

holding constant

$\hat\beta_0 + \hat\beta_2 x_2$: $x_{1,1} = \dots = x_{1,k-1} = 0$, $x_2 = x_2$,

$-\uparrow$

$\hat\beta_0 + \hat\beta_{1,1} + \hat\beta_2 x_2$: $x_{1,1} = 1$, $x_{1,2} = \dots = x_{1,k-1} = 0$, $x_2 = x_2$.

$\hat\beta_{1,1}$: when $x_{1,1} = 1$, compared to baseline $x_{1,0}$,

$\hat{y} \uparrow$ by $\hat\beta_{1,1}$. holding other constant.

3. MLR with interaction.

Suppose x_1 continuous,

x_2 k-level categorical var.

$\downarrow$

$x_{2,1} \dots x_{2,k-1}$ dummy var.

Interaction term between x_1 & x_2 will be $k-1$

$$\hat{y} = \hat\beta_0 + \boxed{\hat\beta_1 x_1} + \boxed{\hat\beta_{2,1} x_{2,1} + \dots + \hat\beta_{2,k-1} x_{2,k-1}}$$ } main

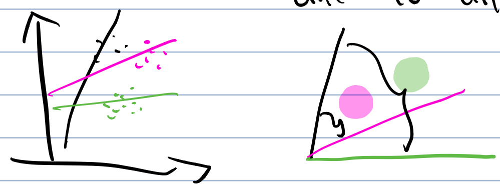
$$+ \hat\beta_{3,1} x_1 x_{2,1} + \dots + \hat\beta_{3,k-1} x_1 x_{2,k-1}$$ $\rightarrow$ interaction. "enhance"

buff if $x_{2,1} = 1$ enhancement.

* $\hat{y} = \hat{\beta}_0 + x_1(\hat{\beta}_1 + \hat{\beta}_{3,1} x_{2,1}) + \cdots$

Interaction : depects $\hat{\beta}_1 +$ ( ) (potential effect)

due to different category



If $x_1 \uparrow$ by 1 unit,

for observations take baseline level. : $\hat{y} \uparrow \hat{\beta}_1$

- - - - - - - - - - level 1 ($x_{2,1} = 1$) : $\hat{y} \uparrow \hat{\beta}_1 + \hat{\beta}_{3,1}$

we say compare to baseline, $\hat{y} \uparrow \hat{\beta}_{3,1}$.

TAKE AWAY :

* Basic : expect to , on average.
* continuous : $x \uparrow 1$ unit. $\Rightarrow \hat{y} \uparrow \hat{\beta}$

  Δx: 1 unit

  factor : obs w/ level 1, compare to baseline level,

  Δx: level compared to baseline $\Rightarrow \hat{y} \uparrow \hat{\beta}_{1,1}$.

* MLR : ,+ hold other constant.
* Interaction : "Buff" + slope.

  Obs w/ baseline level, $x_1 \uparrow 1$ unit $\Rightarrow \hat{y} \uparrow \hat{\beta}_1$

  obs w/ level 1, $x_1 \uparrow 1$ unit $\Rightarrow \hat{y} \uparrow \hat{\beta}_1 + \hat{\beta}_{2,1}$

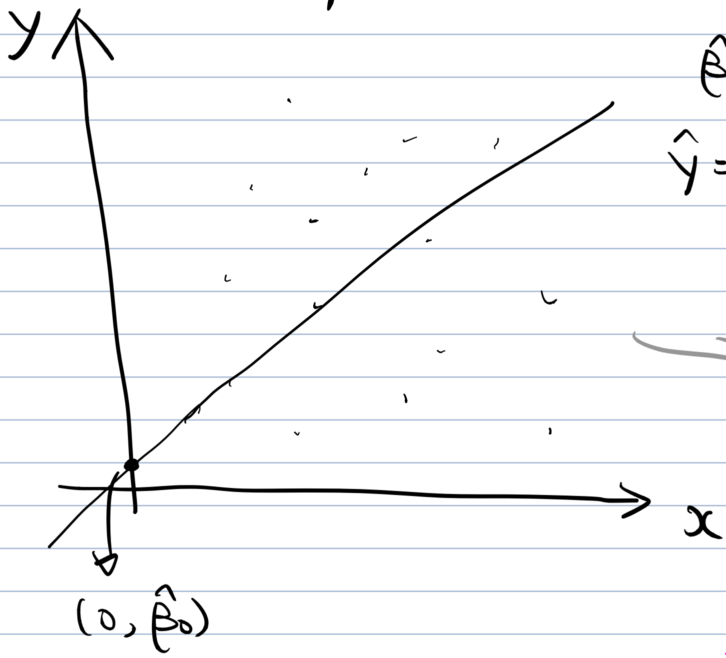
compare to obs w/ baseline level, for obs w/ level 1,

$x_1 \uparrow 1$ unit $\Rightarrow \hat{y} \uparrow \hat{\beta}_{2,1}$

⊛ compare to obs w/ baseline level, for obs w/ level 1,

$\Delta\hat{\beta}_1$ & $\hat{\beta}_{2,1}$

$y \sim 1 + x_1$

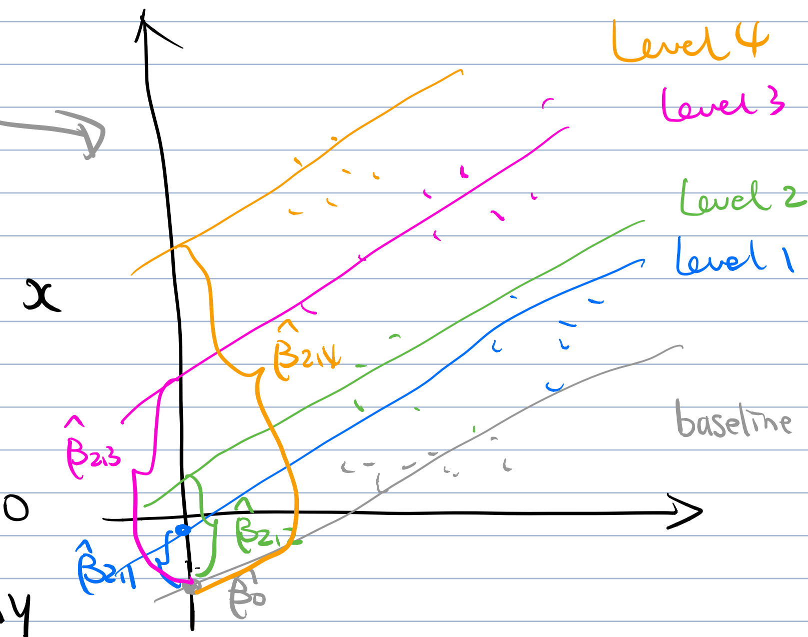


$\hat{\beta}_1$: slope

$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1$

No interaction: buff on intercept

$y \sim 1 + x_1 + x_2$

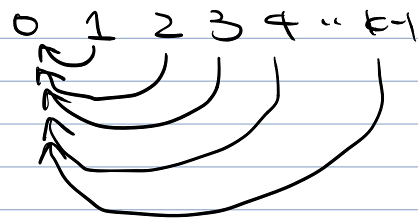


Intercept $\hat{\beta}_0$: All x takes 0

slope $\hat{\beta}_1$: $\Delta x \rightarrow \Delta y$

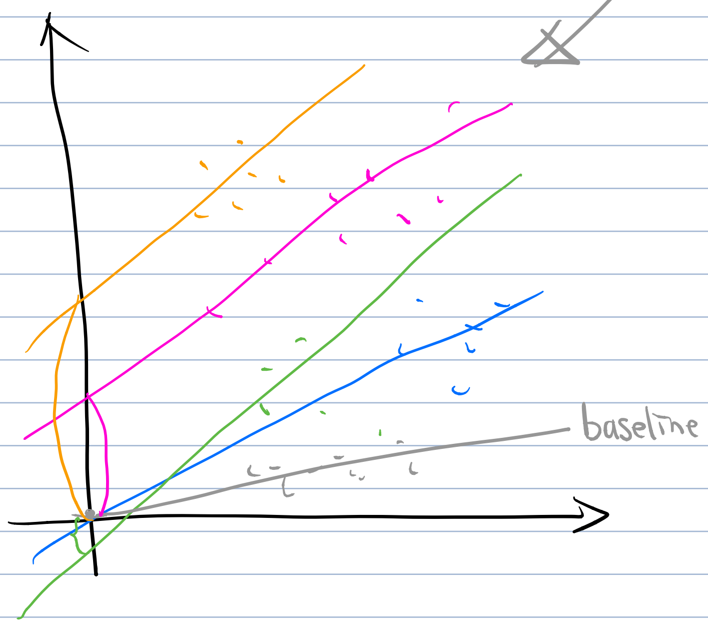
continuous : 1 unit

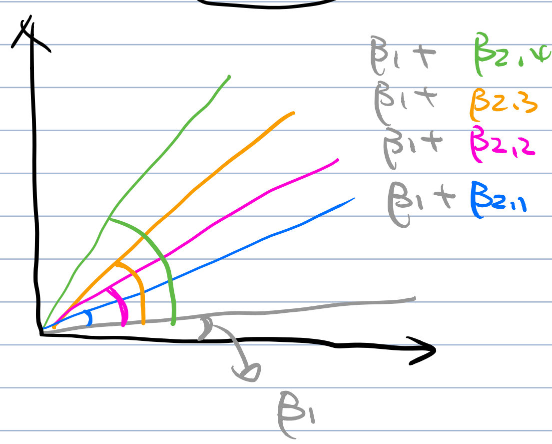
categorical : diff between level.



$y \sim 1 + x_1 + x_2 + x_1 x_2$

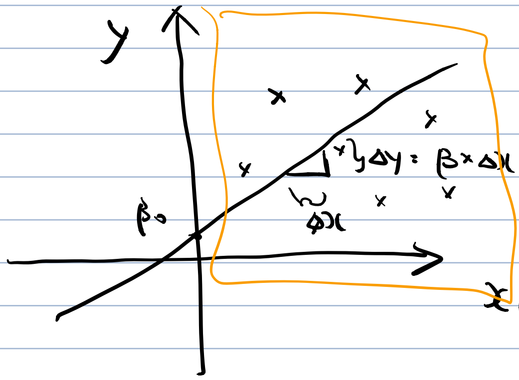
Interaction: buff on slope





$y \sim x_1$

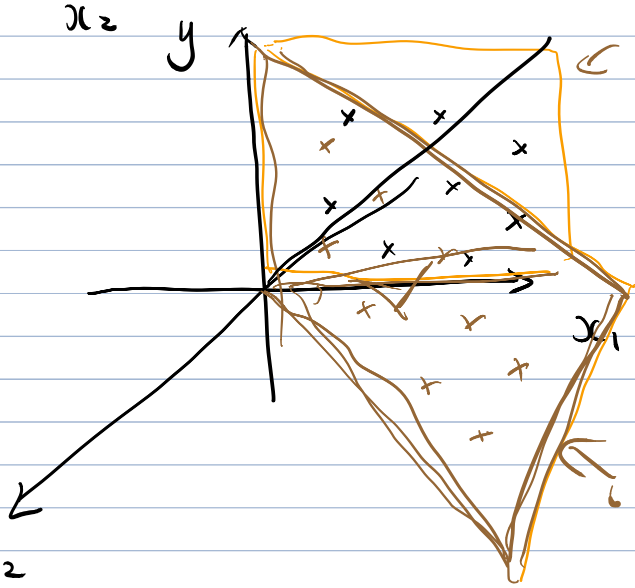
$y \sim x_1 + x_2.$ x_1 and x_2 highly correlated



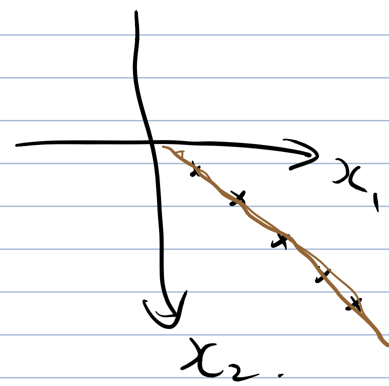
$\beta_1.$

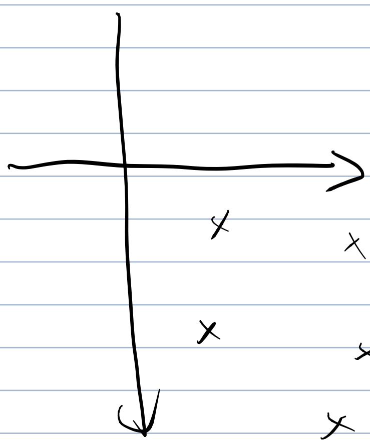
plane.

Add x_2



x_1 & x_2 co-linear.



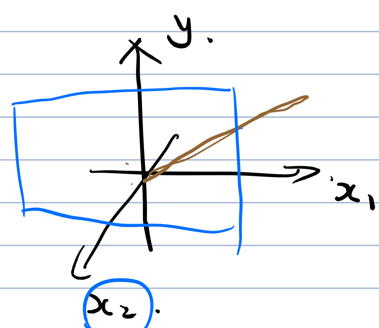


MLR.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2.$$

hold other constant

$\Delta x \to \beta_1 \cdot \Delta y$



$\boxed{x_2 = x_2.}$

Δx