Modeling Neural Population Coordination via a Block Correlation Matrix

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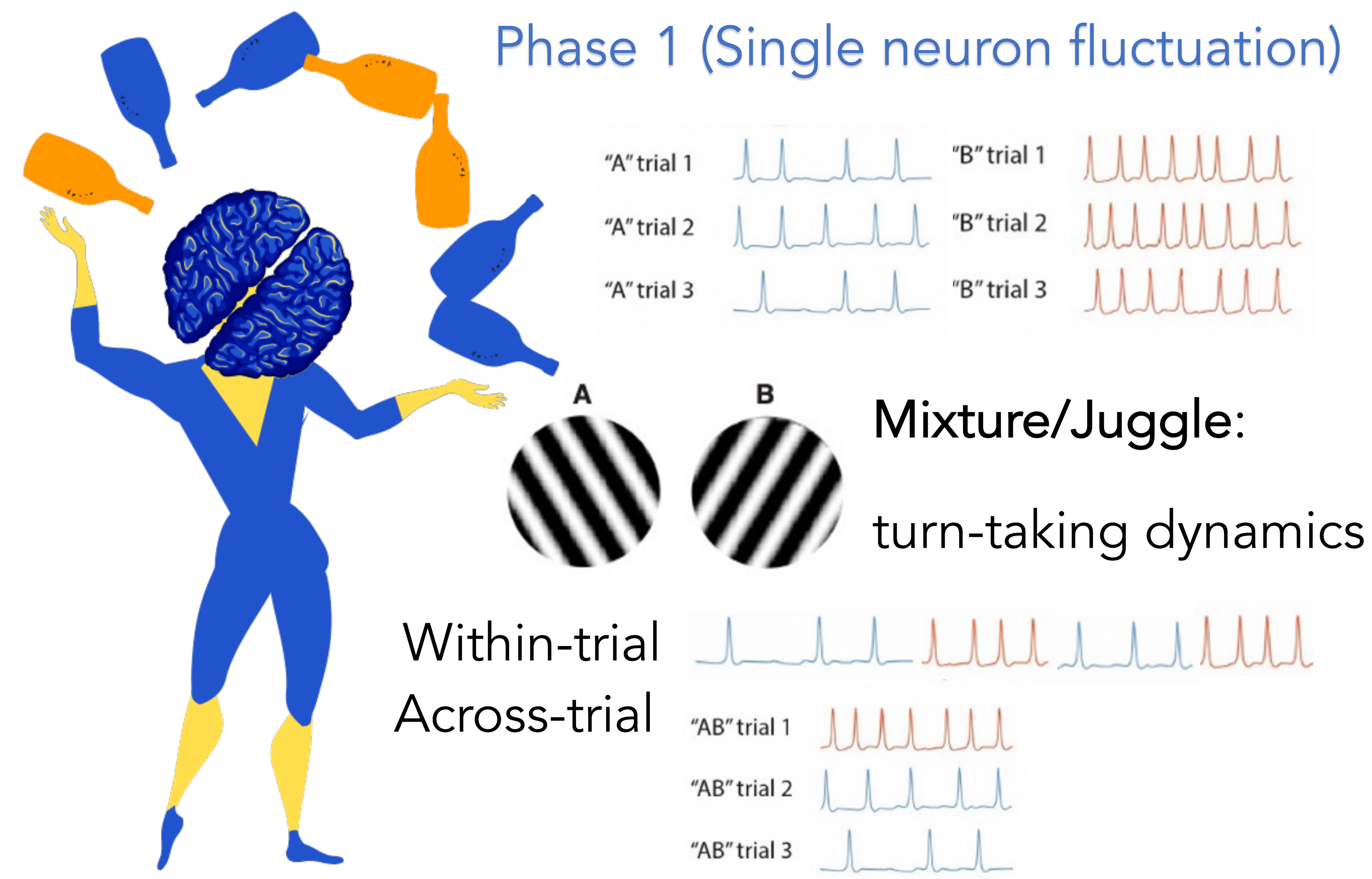


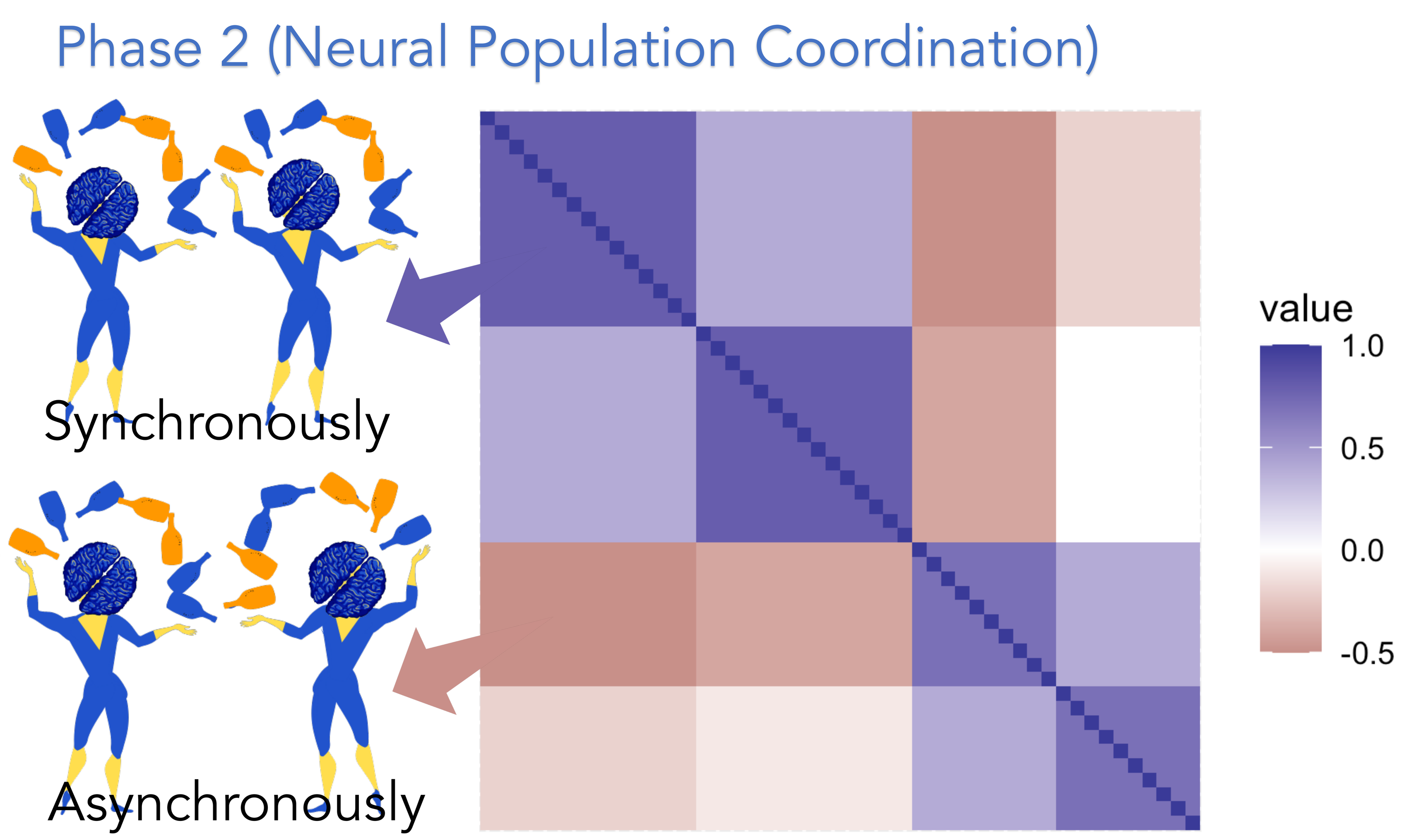


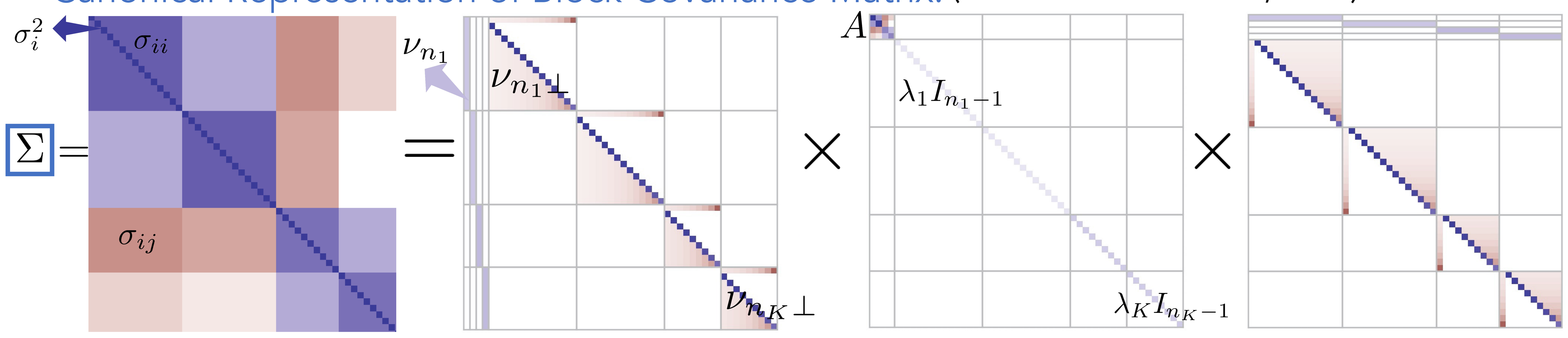
Fig. 1: An example of a block correlation matrix (50 variables)

Research Question

- Block correlation matrix estimation
- Unknown block structure: grouping w.r.t. variables
- Flexibility: off-diagonal correlation ∈ (-1,1)
- Interpretability: model assumptions + priors
- Statistical efficiency: large p small n cases
- Computational efficiency: conjugate priors

Method: Bayesian Block Correlation Matrix Estimation

- Permuted Data: $\tilde{Y} = PY, \quad \tilde{Y} \sim N(0, \Sigma)$
- Canonical Representation of Block Covariance Matrix: (Archakov and Hansen, 2020)



- Correlation Matrix maintains SAME block structure if: $\sigma_i^2 \neq \sigma_{ii}$
- Interpretation: A Representatives $\quad a_{ij} = \sigma_{ij}\sqrt{n_i n_j} \quad a_{ii} = \sigma_i^2 + (n_i - 1)\sigma_{ii}$
  - λ Replicates $\quad \lambda_i = \sigma_i^2 - \sigma_{ii}.$
  - Q Rotation $\quad v_{n_k} = (\frac{1}{\sqrt{n_k}}, \dots, \frac{1}{\sqrt{n_k}})'$ Depends on Group size only
- Log likelihood: $-\frac{NJ}{2}\log 2\pi - \frac{J}{2}\log|A| - \frac{1}{2}\sum_{j=1}^{J}\eta_{j(0)}^{*\prime}A^{-1}\eta_{j(0)}^{*} - \frac{J}{2}\sum_{k=1}^{K}(n_k - 1)\log\lambda_k - \frac{1}{2}\sum_{j=1}^{J}\sum_{k=1}^{K}\eta_{j(k)}^{*\prime}\eta_{j(k)}^{*}/\lambda_k$, where $\eta_j^* = Q'\tilde{Y}_j$,
- Conjugate Priors: $A|\mathcal{C} \sim IW(\nu, A_0), \quad \lambda_k \sim (iid) IG(a_\lambda, b_\lambda).$
- Groups allocation: Mixture of Finite Mixtures (MFM) (Miller and Harrison, 2018)

P : $\mathcal{C}$ denotes partition of $[N]$ induced by $S_1, \dots, S_N$.

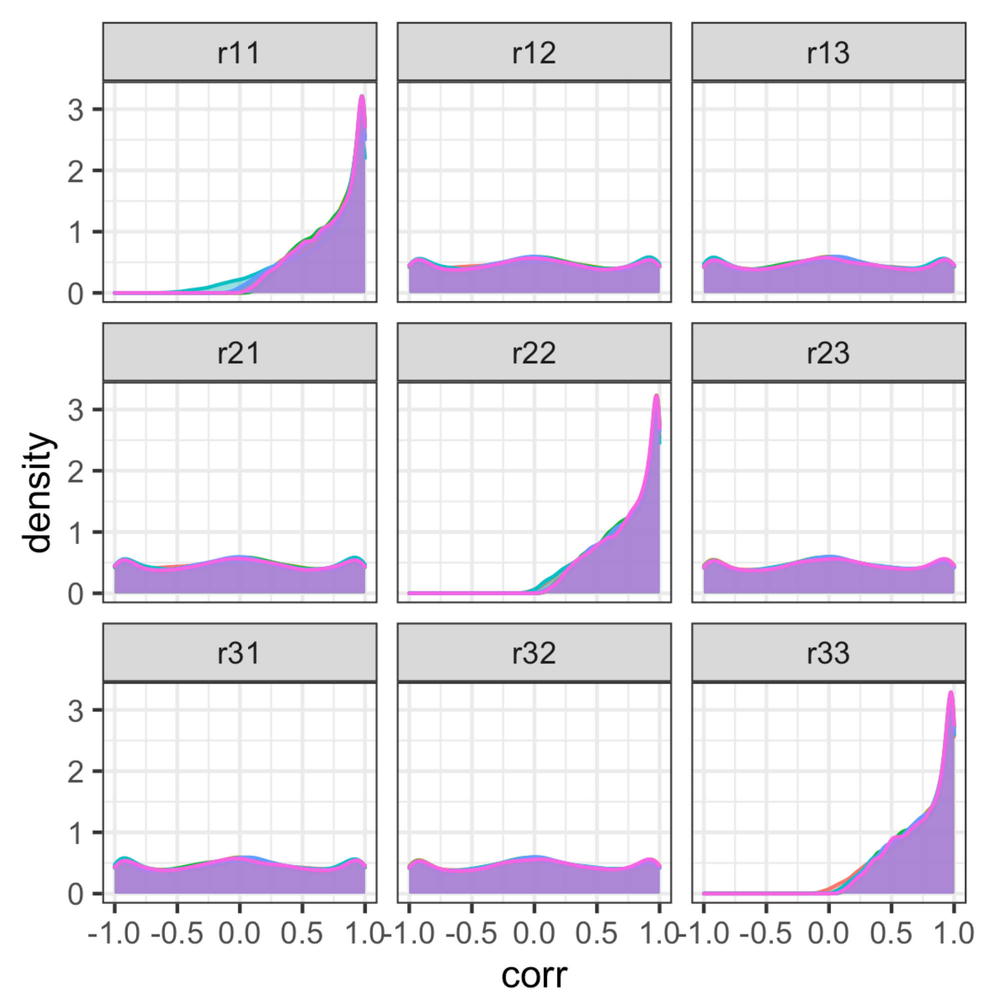
$K \sim p_K$, where p_K is a p.m.f on $\{1, 2, \dots\}$, where we consider $K - 1 \sim \text{Pois}(1)$

Group allocation $S_i \in \{1, \dots, K\}$

$(\pi_1, \dots, \pi_k) \sim \text{Dir}_k(\gamma, \dots, \gamma)$ given $K = k$

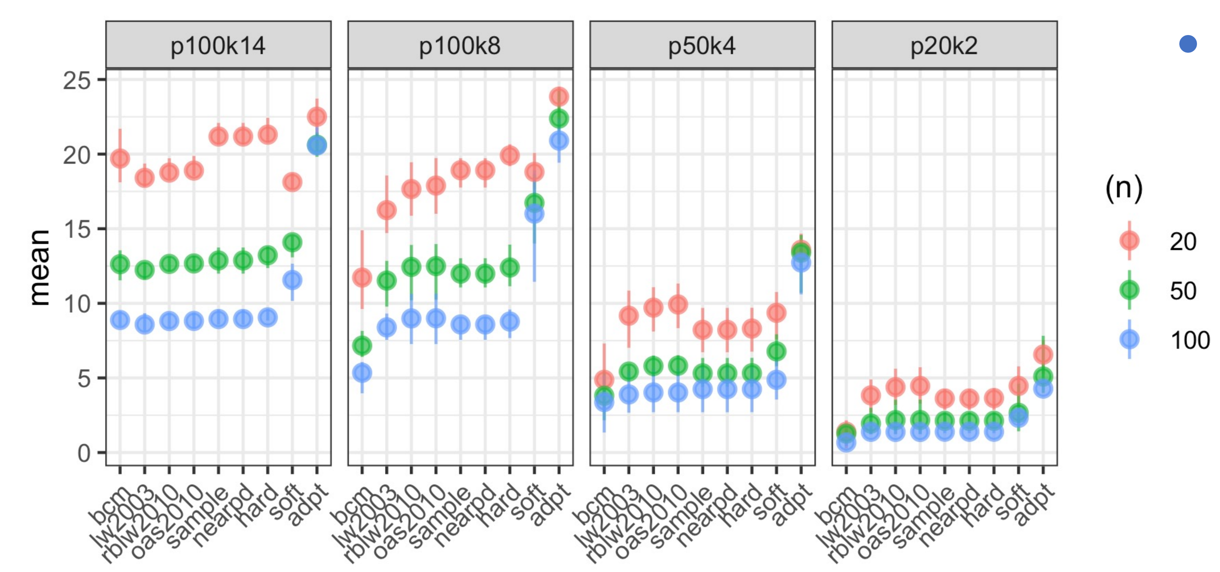
$S_1, \dots, S_N \sim \pi$ (iid) given π

Numerical Experiments

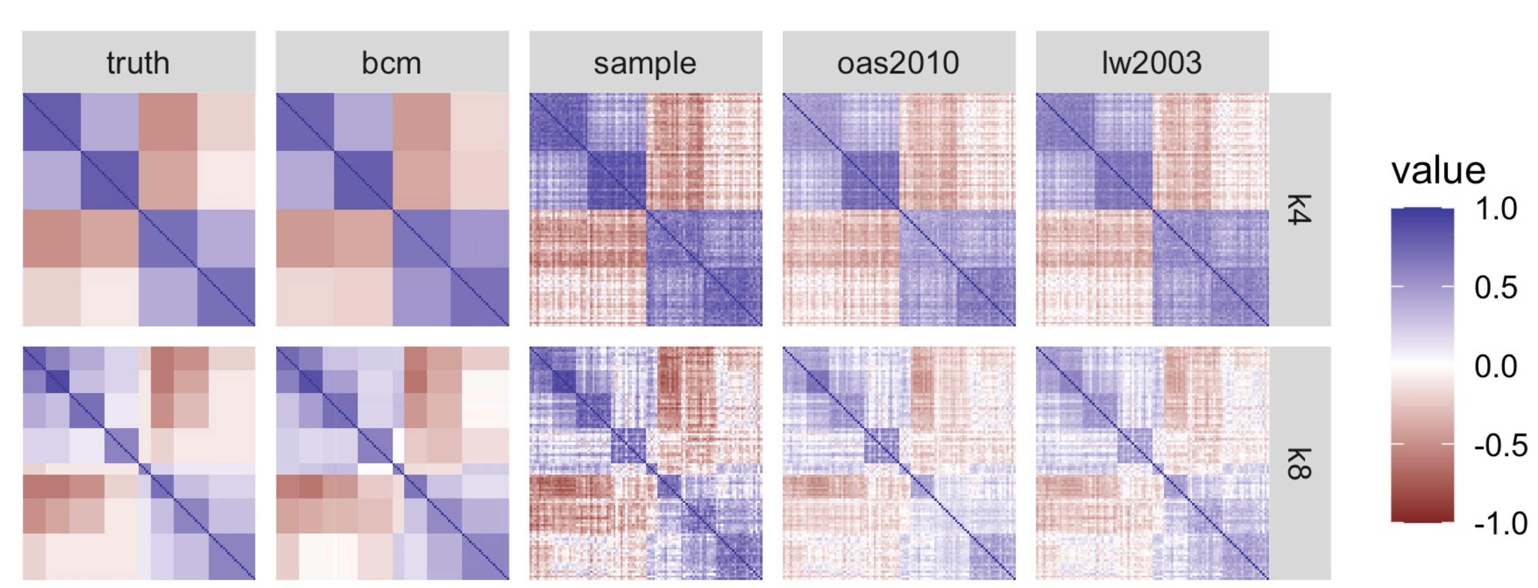


- Non-informative priors:
  (1) invariant to group size
  (2) between-group: uniformly distributed
  (3) within-group: positive, relatively high
- Scale with group size:
  $\nu = K, A_{0(kk)} = n_k,$
  $\lambda_k^{-1} \sim Ga(20, 10)$

Fig. 2: Examples of induced priors for block correlation under different group allocation



- Estimation accuracy:
  BCM outperforms other alternatives except for situations with large p/k ratio (measured by Frobenius Dist)



- Grouping:
  BCM recovers the true block structure in a decent way (smoothing/denoise) even under small n large p cases.
  (p=100, n=20)